Topology Control for Efficient Information Dissemination in Ad-hoc Networks

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Abstract—In this paper, we explore the information dissemination problem in ad-hoc wireless networks. First, we analyze the probability of successful broadcast, assuming: the nodes are uniformly distributed, the available area has a lower bound relative to the total number of nodes, and there is zero knowledge of the overall topology of the network. By showing that the probability of successful broadcast within a bounded number of transmissions is small, we are motivated to extract good graph topologies to minimize the overall transmissions. Three algorithms are used to generate topologies of the network with guaranteed connectivity. These are the minimum radius graph, the relative neighborhood graph and the minimum spanning tree. Our simulation shows that the relative neighborhood graph has certain good graph properties, which makes it suitable for efficient information dissemination.

keywords: topology control, connectivity, transmission power, information dissemination, wireless networks.

I. Introduction

Present day technology enables us to build and deploy wireless nodes to collect and disseminate information for a vast number of applications ranging from space exploration and environmental monitoring to military operations. Parallel to the development of wireless node technology is the increased interest in the study of ad-hoc wireless networks. Unlike a wired network, which typically has a well-defined underlying infrastructure, an ad-hoc wireless network's infrastructure is not known apriori and may in fact change over time [1]. In this work, we are motivated to identify recognizable patterns (i.e. efficient link dependent structure) from a set of randomly placed nodes, such that we can use this information to construct a communication infrastructure with desirable properties. Once the communication topology is defined, we can then solve routing and scheduling problems leveraging this topology.

Conventionally, the topology of an ad-hoc network can be defined by the transmission power of each node. If the power is too low, we cannot guarantee connectivity of all the nodes. If the power is too high, there could be too much interference (i.e. multi-user interference may not allow for efficient use of bandwidth per transmission power.). In a sensor network where each node has limited battery power (possibly rechargeable), the objectives are:

- good energy-per-bit performance
- reliability

We aim for a topology which guarantees connectivity while minimizing power and interference. In particular, we consider networks where minimizing delay is less important than ensuring good energy-per-bit performance. Related work considering other objectives can be found in [2], [3], [4].

To determine the suitability of the topologies examined, we consider information dissemination (broadcast) on these topologies. Information dissemination is needed in cooperative sensor networks [5] and cooperative modulation techniques [6], where the main objective is to prolong the life time of sensor nodes.

We consider the topology control problem from two complementary approaches. In the first approach, we consider minimizing interference by imposing a spacial constraint on the node locations while assuming a fixed transmission radius for all the nodes. However, our analysis shows that this constraint has a negative effect on information dissemination. As the number of nodes and the area increase, the probability of successful broadcast within a constant number of transmissions decreases drastically as the network becomes sparser. In the second approach, we consider a proximity graph called the Relative Neighborhood Graph (RNG). RNGs are concerned with extracting the shapes and structures of point sets, and have been successfully used in many areas such as archeology [7], biology [8], pattern recognition [9], and morphological properties of empirical networks [10]. We examine the RNG as a viable means of achieving good performance for constructing a back-bone communication architecture for a set of randomly placed nodes. From our simulation, RNG displays desirable properties in terms of average transmission radius (related to power), average node degree (related to interference), and reliability (fault-tolerant).

II. Topology Control by Spacial Constraint

Given n nodes such that each node covers a RF circular area $\pi d_{RF}^2 = \frac{\log n + c(n)}{n}$, Gupta and Kumar showed that network connectivity approaches probability 1 as $c(n)$ approaches infinity, where $c(n)$ is the connectivity measure defined in [11]. Intuitively, as the number of nodes increases, the connectivity of the network also increases.

To minimize interference, we impose a spacial constraint

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on the plane A_n containing n nodes, with respect to a common transmission radius d. Since the nodes are randomly placed in A_n with uniform distribution, we can decrease interference by making A large enough so that the likelihood of nodes interfering with each other is small. We then analyze the probability of successful broadcast from a source node, using a bounded number of transmissions.

Let $l_i, l_j \in \mathcal{R}^2$ be the locations of nodes v_i and v_j respectively, where $v_i \neq v_j$. Direct connectivity between any pair of nodes v_i and v_j is defined by the transmission radius d. Specifically, for v_i and v_j, we have $\| l_i - l_j \| \leq d$, where the norm used is the Euclidean norm (i.e., L^2-norm). We say that v_i and v_j have multi-hop connectivity if there is a non-empty set of nodes P such that information can be routed between v_i and v_j through the nodes of P.

Given n nodes, let $l_{src}, l_1, l_2, \ldots, l_{n-1} \in \mathcal{R}^2$ be the locations of the source node and nodes $v_1, v_2, \ldots, v_{n-1}$ respectively, and let V contain all the nodes. Let $\mathcal{N}(l_i)$ be the maximal set of nodes contained in the circle of radius d centered at l_i. Specifically, we have $\mathcal{N}(l_i) = \{v_z : v_z \in V$ and $\|l_i - l_z\| \leq d\}$.

Assume each node has a common transmission radius of d, the area covered by a node v_i is πd^2 centered at l_i. We use $A(v_i)$ to denote the circular area covered by v_i. Suppose nodes v_i and v_j are directly connected. Ignoring edge effects, the maximum area where a third node v_k can reside such that v_k is directly connected to v_j but not to v_i, is upper bounded by $A(v_j) - [A(v_i) \cap A(v_j)]$. Let $\alpha = 2\pi + 3\sqrt{3}$.

Lemma 1: Suppose there exists nodes v_i, v_j at location $l_i, l_j \in \mathcal{R}^2$ respectively, such that $\|l_i - l_j\| = d$. If each node can cover an area with radius d, the non-overlapping area of either node v_i or v_j is $\frac{\alpha d^2}{6}$.

Proof of Lemma 1 : Consider Figure 1, where two nodes are separated by distance d, and RF radial transmission distance of nodes v_i and v_j are d_i and d_j respectively, where $d_i = d_j = d$.

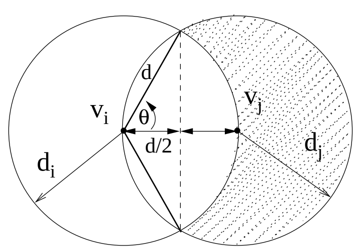

Fig. 1. Non-overlapping area (shaded) with respect to Node v_j.

Each node covers an area of πd^2. Since l_i (Node v_i) and l_j (Node v_j) are exactly d apart, then there exists a perpendicular line between l_i and l_j such that it bisects the line joining l_i and l_j. Thus, we can compute θ in Figure 1 as $\theta = \cos^{-1}(\frac{d/2}{d}) = \cos^{-1}(1/2) = \pi/3$. Since the angle representing the overlap is twice the angle θ, the area encompassed by the arc of the two points is $\frac{1}{2}2\theta d^2 = \frac{\pi d^2}{3}$. Subtracting the triangular area encompassed by the arc, we have $\Delta = \frac{\pi d^2}{3} - 2 \cdot \frac{1}{2}(\sqrt{3}\frac{d}{2})(\frac{d}{2}) = \frac{\pi d^2}{3} - \frac{\sqrt{3}d^2}{4} = \frac{(4\pi - 3\sqrt{3})d^2}{12}$.

Since the overlap occurs on both sides of the perpendicular line bisecting the line joining l_i and l_j, we have $2\Delta = \frac{(4\pi - 3\sqrt{3})d^2}{6}$. To obtain the non-overlapping (shaded) area $A_{non}(d)$, we have

$$\begin{aligned} A_{non}(d) &= \pi d^2 - 2\Delta \\ &= \pi d^2 - \frac{(4\pi - 3\sqrt{3})d^2}{6} \\ &= \frac{\alpha\, d^2}{6}\ . \end{aligned}$$

□

To impose a spacial constraint, we compute A_n as follows. The first node is placed in a circle of $\pi\, d^2$, where the source node is placed in the center of the circle. The next $n-2$ nodes are scattered over a total area of $(n-2)\, \frac{\alpha\, d^2}{6}$ to avoid interference. We do not need to consider additional space for the last node. Thus, we select

$$A_n \quad \geq \quad \pi\, d^2 + \frac{(n-2)\, \alpha\, d^2}{6}\ , \tag{1}$$

as a lower bound on the spacial placements of nodes.

We now analyze the total number of transmissions required for the broadcast of a single bit, which is topology independent and without any knowledge of the topology. Let T_{src} be the number of transmissions required to broadcast error free in a multi-hop manner from the source node.

Theorem 2: Consider n nodes with broadcast radius d randomly placed over area A_n with uniform distribution. The upper bound on the probability of a node requiring $T = k$ transmissions to propagate a bit of information to all other $n-1$ nodes is upper bounded by $G(k,n)(d^2/A_n)^{n-1}$, where

$$G(k,n) = \frac{6\pi(2\pi + 3\sqrt{3})^{k-1}[4\pi - 3\sqrt{3} + \alpha k]^{n-1-k}}{6^{n-1}}\ ;$$

using (1), we have

$$Pr\{T_{src}(n) = k\} \quad \leq \quad \frac{6\pi\alpha^{k-1}[4\pi - 3\sqrt{3} + \alpha k]^{n-1-k}}{(6\pi + \alpha(n-2))^{n-1}}\ .$$

Proof of Theorem 2:
Let l_{src} be the location of the source node. Let l_i be the location of v_i, where $1 \leq i \leq n-1$. For the broadcast of a bit from the source requiring a single transmission to another node v_i, the receiving node must be within radius distance d of the source node, and so the probability of a single transmission to a single node, $Pr\{T_{src}(1) = 1\}$, we have

$$\begin{aligned} Pr\{T_{src}(1) = 1\} &= Pr\{l_i \in A(l_{src})\} \\ &= \frac{\pi d^2}{A_n} \triangleq p_{first}\ . \end{aligned} \tag{2}$$

The probability of a single transmission from the source node to all other $n-1$ nodes is p_{first}^{n-1}. For nodes to require multiple transmissions, the spatial allowable area not in contact with any other nodes, and thus requiring more transmissions, is upper bounded by the non-overlapping area between two nodes w.r.t. one of the nodes. Thus, the probability of broadcasting a bit over any transmission other than the first transmission, p_{trans} can be written as the probability of the source node propagating through v_i to transmit a bit to v_j such that v_j is at a distance larger than d. The number of transmissions is lower bounded by the number of hops. Thus, using Lemma 1, we have

$$\begin{aligned} p_{trans} &\leq Pr\{l_i \in \mathcal{N}(l_{src}) \bigcap l_j \in \mathcal{N}(l_i)/l_j \notin \mathcal{N}(l_{src})\} \\ &= \frac{\alpha d^2}{6A_n} \,. \end{aligned} \tag{3}$$

For a total of n nodes where the source node needs to propagate over i transmissions to all of the other $n-1$ nodes, there are $n-1-i$ nodes that are allowed to placed anywhere within the allowable area of transmission. Thus, the probability of the $n-1-i$ nodes residing within the transmission area has an upper bound of

$$\begin{aligned} p_{others}(i) &= \frac{1}{A_n}\left(\pi\, d^2 + \frac{(i-1)\,\alpha\, d^2}{6}\right) \\ &= \frac{[4\pi - 3\sqrt{3} + \alpha i]d^2}{6A_n}. \end{aligned} \tag{4}$$

Using (2), (3), and (4), we can upper bound the legitimate area required in order to transmit $T_{src}(n) = k$ as the probability of at least one node contained in the initial space, at least one node contained in each of the non-overlapping spaces (equivalent to the $k-1$ transmissions), and all other nodes contained anywhere among the allowable space. Thus, we have

$$\begin{aligned} Pr\{T_{src}(n) = k\} &\leq p_{first} \cdot p_{trans}^{k-1} \cdot p_{others}(k)^{n-1-k} \\ &= \frac{\pi d^2}{A_n} \cdot \left(\frac{\alpha d^2}{6A_n}\right)^{k-1} \\ &\quad \cdot \left(\frac{[4\pi - 3\sqrt{3} + \alpha k]d^2}{6A_n}\right)^{n-1-k} \\ &= G(k,n)\frac{d^{2(n-1)}}{A_n^{n-1}} \,. \end{aligned} \tag{5}$$

For space constraints, let $F(\alpha, k) = 4\pi - 3\sqrt{3} + \alpha k$. Combining (5) and (1), we obtain

$$\begin{aligned} Pr\{T_h(n) = k\} &\leq \frac{6\pi\alpha^{k-1}F(\alpha,k)^{n-1-k}d^{2(n-1)}}{6^{n-1}A_n^{n-1}} \\ &\leq \frac{6\pi\alpha^{k-1}F(\alpha,k)^{n-1-k}d^{2(n-1)}6^{n-1}}{6^{n-1}[(6\pi + \alpha(n-2))d^2]^{n-1}} \\ &= \frac{6\pi\alpha^{k-1}F(\alpha,k)^{n-1-k}}{(6\pi + \alpha(n-2))^{n-1}} \\ &= \frac{6\pi\alpha^{k-1}[4\pi - 3\sqrt{3} + \alpha k]^{n-1-k}}{(6\pi + \alpha(n-2))^{n-1}} \,, \end{aligned}$$

proving Theorem 2. $\square$

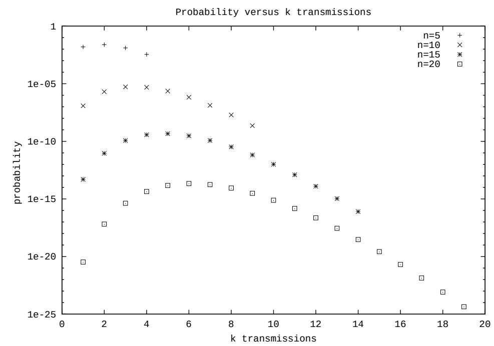

Fig. 2. Probability of number of transmissions for n independent uniformly spaced nodes.

As the above analysis of Theorem 2 and plot of Figure 2 shows, the upper bound probability of successful information dissemination with a fixed number of transmissions is small given a lower bound constraint on the allowable area of uniformly placing nodes. In other words, to minimize interference by imposing a spacial constraint does not give us a communication graph suitable for information dissemination. This discovery justifies our next approach where we look for desirable structural properties inherent in the node placements, and we use these properties to select a communication topology for efficient information dissemination.

III. Topology Control by Proximity Graphs

For the purpose of information dissemination, we must guarantee a connected communication topology. In this initial study, we propose three classes of topologies which meet the connectivity requirement and we compare the graph properties of these by simulation. These graphs are: (1) minimum radius, (2) relative neighborhood graph and (3) minimum spanning tree. We will discuss each class of graphs in more detail in forthcoming subsections. The graph properties we are interested in are:

- **radius**: proportional to power over data rate.
- **hop diameter**: the number of hops (network diameter) can be used as a lower bound for the number of transmissions.
- **edge density**: fewer edges simplifies the global scheduling.
- **node degree**: affects the number of time slots (or frequencies) needed in local scheduling, i.e., non-interfering nodes can use the same time slots (frequency) for local communication.
- **number of biconnected components**: this shows the number of weak points within the network.
- **size of largest biconnected component**: used to measure the network robustness.

A. *Minimum Radius (minR)*

In this class of graphs, we assume the nodes have a common transmission radius d. We then compute the smallest value for d which guarantees connectivity. Let A denote the square area containing the nodes. A side of A has length $\sqrt{A}$. The algorithm iteratively performs a binary search for the smallest d. In each iteration, the algorithm computes a graph with transmission radius d and checks if the graph is connected. If the graph is connected, then d is decreased, otherwise, d is increased. The algorithm proceeds in iterations until we find the smallest d such that, using d, the communication graph is connected but when using $d-1$, the communication graph is partitioned. For simplicity of simulation, we assume the nodes' coordinates and d are integers. In Figure 3, we show a minR graph with 50 nodes.

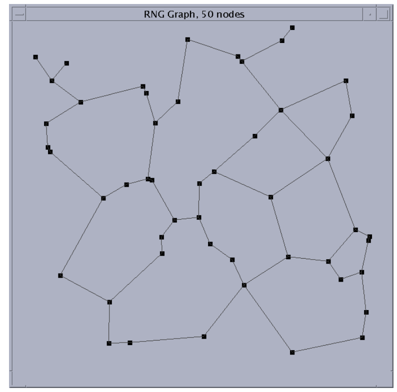

Fig. 4. Relative Neighborhood Graph with 50 nodes

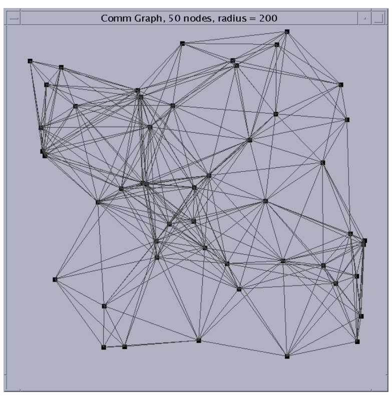

Fig. 3. Minimum (fixed) Radius Graph with 50 nodes

B. *Relative Neighborhood Graph*

The relative neighborhood graph (RNG) of a node set V in Euclidean space is the graph $G = (V, E)$, where $(v_i, v_j) \in E$ if and only if there is no node $v_z \in V$ such that $\| l_i - l_z \| < \| l_i - l_j \|$ and $\| l_j - l_z \| < \| l_i - l_j \|$, or equivalently, the edge between nodes v_i and v_j is valid if there does not exists any node closer to both v_i and v_j. Referring back to Figure 1, a radius of $\| l_i - l_j \|$ is used for the pair of nodes v_i and v_j. Note that, in RNG, a different radius may be used for each pair of nodes, and so for Figure 1, we could have $d_i \neq d_j$. If the intersection of $A(v_i)$ and $A(v_j)$ does not contain any other nodes, then Node v_i and Node v_j are relative neighbors (i.e. they are directly connected). Figure 4 shows a RNG with 50 nodes.

C. *Minimum Spanning Tree*

A minimum spanning tree is a tree connecting all the nodes such that the total edge length is minimized. Since the minimum spanning tree (MST) is a subgraph of RNG, we use RNG in the computation of MST. Note that RNG is a subgraph of the Delaunay triangulation, and the Delaunay triangulation is a planar graph. Thus, the number of edges in RNG is bounded by $3n - 6$ *. We then only need to examine $O(n)$ edges for inclusion/exclusion in the MST. An example of a 50 node MST is shown in Figure 5.

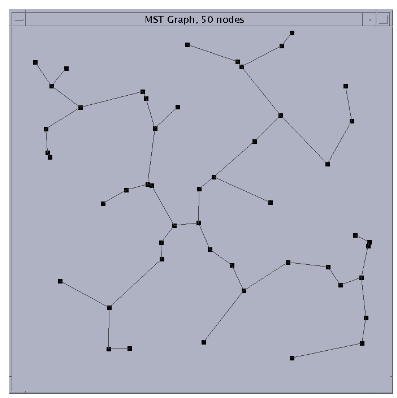

Fig. 5. Minimum Spanning Tree with 50 nodes

D. *Algorithms*

In our simulation, we use sequential algorithms (single processor) to compute the graphs and then compare the graph properties of the produced topologies. The sequential minR algorithm has a computational cost of $O(n^2 \log \sqrt{A})$ because there can be at most $\log \sqrt{A}$ iterations, where each iteration checks n^2 node pairs.

A brute-force sequential RNG algorithm has a computational cost of $O(n^3)$, by checking each node with each of

*This is a corollary of Euler's polyhedra formula which was written in a letter from Euler to Goldbach in 1750.

the n^2 possible edges. However, this can be improved by using a pre-computed Delaunay triangulation (DT). Since RNG is a subgraph of DT, it suffices to check each node against each of the DT edges. This checking step takes $O(n^2)$ time since DT has $O(n)$ edges. The pre-computation of DT takes $O(n \log n)$ time [12].

The sequential MST algorithm makes use of the previously computed RNG. The algorithm first sorts the edges of RNG with respect to the edge length, from shortest to longest. This takes $O(n \log n)$ operations. An edge is included in MST if it does not create a cycle in the graph. This is performed by using disjoint sets. Nodes that are connected are placed in the same set. If the tested edge connects two nodes belonging to different sets, then the edge is added to MST and we take the union of the two sets. If the tested edge connects two nodes belonging to the same set, then this edge creates a cycle and it is rejected. Using the union-find data structure, the number of operations for computing MST is bounded by $O(n \log n)$ when the RNG is given.

In this paper, our goal is to compare the graph properties of minR, RNG and MST. Hence, we implemented the algorithms on the sequential model only. From our simulation, RNG compares favorably with minR and MST (see Table I). We compare the quality of each topology with respect to each metric. Table I summarizes the result obtained by generating 1000 different placements of 800 nodes each. In the table, a score between 0 and 100 is assigned to each topology considering its performance with respect to the specific metric. The best performance gets 100 and the worst gets 0. Other scores are assigned accordingly in reference to the best and worst performances.

performance score	*minR*	*MST*	RNG
hop diameter	100	0	65
average power	0	100	75
average node degree	0	100	96
num. biconnected components	100	0	95
biconnected component size	100	0	94

TABLE I

PERFORMANCE COMPARISONS FOR MINR, MST, RNG

To make this result more practical, we designed a distributed algorithm for RNG in a separate paper[13]. The distributed algorithm assumes that each node has an omni-directional antenna for communication. Each node can sense the direction of incoming signals and that transmit power can be controlled between zero and maximum power. Assuming that at least one node is awake initially, all the nodes will become awake in $O(D)$ time, where D is the diameter of the RNG. After each node is awake, it takes constant time for each node to compute RNG locally. That is, each node only need to determine the RNG edges connecting itself with other nodes.

E. Simulation Results

For our simulation† runs, we generated n nodes, randomly placed in an area A, where $5 \leq n \leq 800$, and A is a fixed area of 600^2 units^2, and diagonal $600\sqrt{2}$ units. Two uniformly distributed random integers are generated as the coordinate of each node. For each n, we make 1000 runs. In each run, we use the same set of nodes for the computation of the minimum radius graph (minR), the relative neighborhood graph (RNG) and the minimum spanning tree (MST).

From the simulation, we observe the following:

• **radius**: In minR, every node is required to use the same radius d; thus, d is the smallest radius which renders a connected graph. For RNG and MST, the radius is the longest edge (in Euclidean distance) in the graphs, assuming different transmission radii were possible. Figure 6 shows

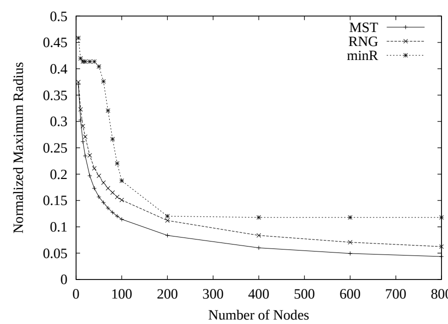

Fig. 6. Maximum transmission radius, averaged over 1000 runs.

the plot of the average maximum radius with respect to the number of nodes, normalized with respect to the diameter of A which is $600\sqrt{2}$. As expected, the radius decreases as the number of nodes increases for all three graphs. On the average, MST requires a smaller radius than RNG, where RNG requires a smaller radius than minR. Note that as n increases, the performance of RNG is closer to MST than it is to minR.

• **hop diameter**: the hop diameter of a network is the maximum number of hops among the shortest paths connecting any pair of nodes. This can be used as a lower bound for the number of transmissions required for broadcast. Therefore, it is important to obtain a topology which minimizes the hop diameter. Note that, a partitioned network has hop diameter $+\infty$. Figure 7 shows the plot of hop diameters with respect to the number of nodes. On the average, minR has the lowest hop diameter and MST has the highest. It is worth noting that the RNG hop diameter is closer to the minR than it is to the MST, which means RNG is almost as good as minR in this respect.

• **edge density**: the edge density of a graph is computed relative to the maximum number of all possible edges. A graph with n nodes can have at most $\frac{n\,(n-1)}{2}$ edges. Let

†We have implemented the sequential algorithms in JAVA (version 1.2) on a Sun Ultra-10 workstation.

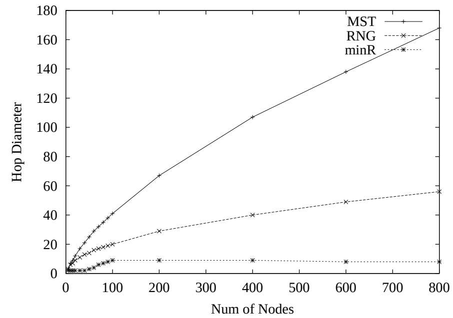

Fig. 7. hop diameter, averaged over 1000 runs.

this number be *maxE*. The density of a graph $G = (V, E)$ is defined to be $|E|/maxE$, where density is a real number between 0 and 1. We can then compare the densities of MST, RNG and minR. From Figure 8, we observe that

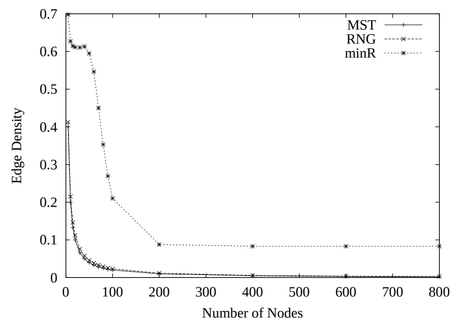

Fig. 8. edge density, normalized and averaged over 1000 runs.

the edge densities of both MST and RNG are very low. As a matter of fact, the plots of MST and RNG almost coincide with each other. In Figure 8, minR has a higher edge density, however, it also decreases very fast as the number of node increases. This is to be expected because as n increases, the number of all possible edges increases quadratically. On the other hand, as n increases, the radius in minR decreases (Figure 6), resulting in fewer edges. Thus, the edge density decreases. A lower edge density may lead to a shorter transmission schedule.

• **node degree**: the node degree is the number of neighbors having direct communication with the node. A lower node degree implies a lower interference with neighbors, if a node may only communicate with directly connected neighbors while other nodes in the neighborhood could be turned off. The node degree affects the scheduling of transmissions. A higher node degree implies that a longer schedule is needed. For each graph, we find the node with the highest node degree, defined as the maximum degree of the graph. In Figure 9, RNG and MST have low node degrees compared to minR. As n increases, the maximum node degree in MST and RNG approaches a small constant. On the other hand, the maximum node degree of minR appears to increase linearly with respect to n. This makes RNG and

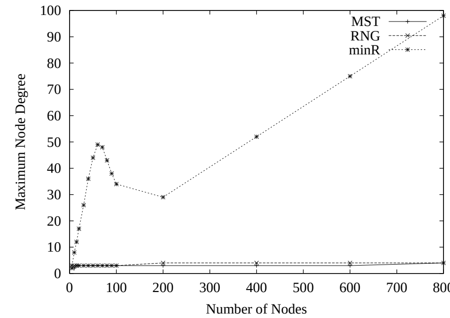

Fig. 9. maximum node degree, averaged over 1000 runs.

MST more scalable when local scheduling is used.

• **number of biconnected components**: a biconnected component is a maximal subgraph of a connected graph such that the deletion of any node does not disconnect the subgraph. The number of biconnected components reveals the number of weak points within the network topology. Since biconnected components are connected by articulation points whose failure results in a partitioned network, fewer biconnected components implies a more fault-tolerant network. Since MST is a tree, it does not contain

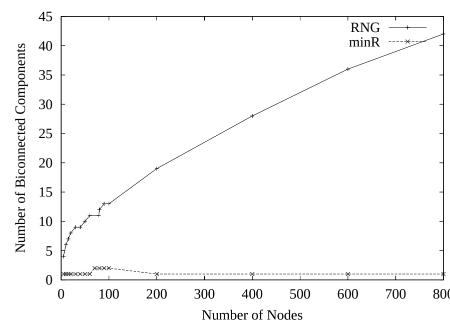

Fig. 10. number of biconnected components, averaged over 1000 runs.

biconnected components (unless we consider each node to be biconnected with itself). Figure 10 shows that minR has fewer biconnected components than RNG. To clarify, in the sparse sub-graphs of RNG, each sub-graph may have a tree topology. In that case, each node is counted as a single biconnected component. This may explain why the number of biconnected components in RNG seems to be much higher, compared to minR.

• **largest biconnected component size**: a connected graph may contain several biconnected components. The largest biconnected component is the one containing the most number of nodes. The size of the component is the number of nodes it contains. By examining the largest biconnected component, we can determine what percentage of the nodes are not biconnected with the majority of nodes. If the largest biconnected component contains 90% of the nodes, then even if the number of biconnected components is high, we are guaranteed that 90% of the network is fault-tolerant. Figure 11 shows that for most n values, the largest

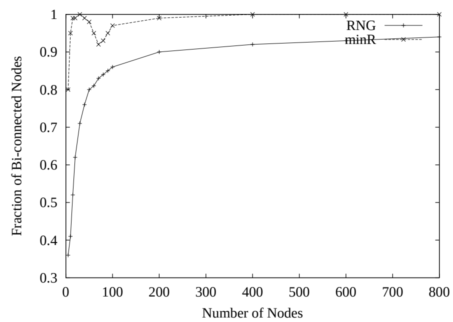

Fig. 11. fraction of nodes in largest biconnected component, normalized and averaged over 1000 runs.

biconnected component in minR contains over 90% of the nodes. The performance of RNG is not far behind with 86% at $n = 100$ and over 90% for $n \geq 200$.

IV. Conclusion

The desire to efficiently reduce the overall energy-per-bit of a node and the analysis indicating the low probability of a bit of information successfully being disseminated among the other nodes with zero knowledge of network topology motivated our study into the graph connectivity for reducing overall transmissions.

From the simulation results, we motivate minR, RNG and MST as follows. To guarantee connectivity, we need a spanning tree at the least. MST is a good choice because it strives to minimize power for fixed data rate. The MST is good in terms of the average maximum transmission radius, edge density and maximum node degree. However, the MST is not fault-tolerant because any node or edge failure will partition the network. It then makes sense to look at a super-graph of MST which still has some of the good graph properties of the MST. For this, we proposed the RNG. The RNG also is good in terms of transmission radius, edge density, and maximum node degree. In addition, our simulation shows that for $n \geq 100$ (or the node density $\geq \frac{n}{A} = \frac{100}{600^2} = \frac{1}{3600}$), the largest biconnected component of RNG contains at least 86% of the nodes. Although this is not as good as minR, it is close. The RNG may have a higher number of biconnected components. However, since RNG's largest biconnected component contains the majority of nodes, this offsets the importance of the number of biconnected components. Concerning hop diameter, RNG is better than MST and worse than minR. However, RNG's hop diameter is closer to minR than it is to MST. The hop diameter can be used as a measure for information dissemination. While the worst case analysis indicates that $O(n)$ transmissions might be needed for broadcasting one bit of information, on the average, the number of transmissions needed may be closer to hop diameter than to n. The minR is good in terms of the hop diameter, the number of biconnected components and the average largest biconnected component size. However, minR's disadvantages are the higher transmission radius, higher edge density and a node degree which increases linearly with respect to the number of nodes. In light of the above, we suggest that RNG can be a good candidate to consider as a target topology for communication in terms of minimizing power and interference while guaranteeing connectivity (and biconnectivity for the majority of the nodes). From the simulation, RNG shows good graph properties when compared with minR and MST.

For future work, we propose to investigate other topologies related to RNG, RNG's implications on amplifiers, and how to extend proximity graphs to model environments with obstacles blocking line-of-sight between nodes.

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